A. Height of Potential Barrier Across *p-n* Junction

Consider a *p-n* junction whose *p*-side is doped uniformly with concentration N_a and the *n*-side with a uniform density N_d. Let the junction be abrupt i.e., there is an abrupt change from *p*-type impurity to *n*-type impurity. The variation of space charge density

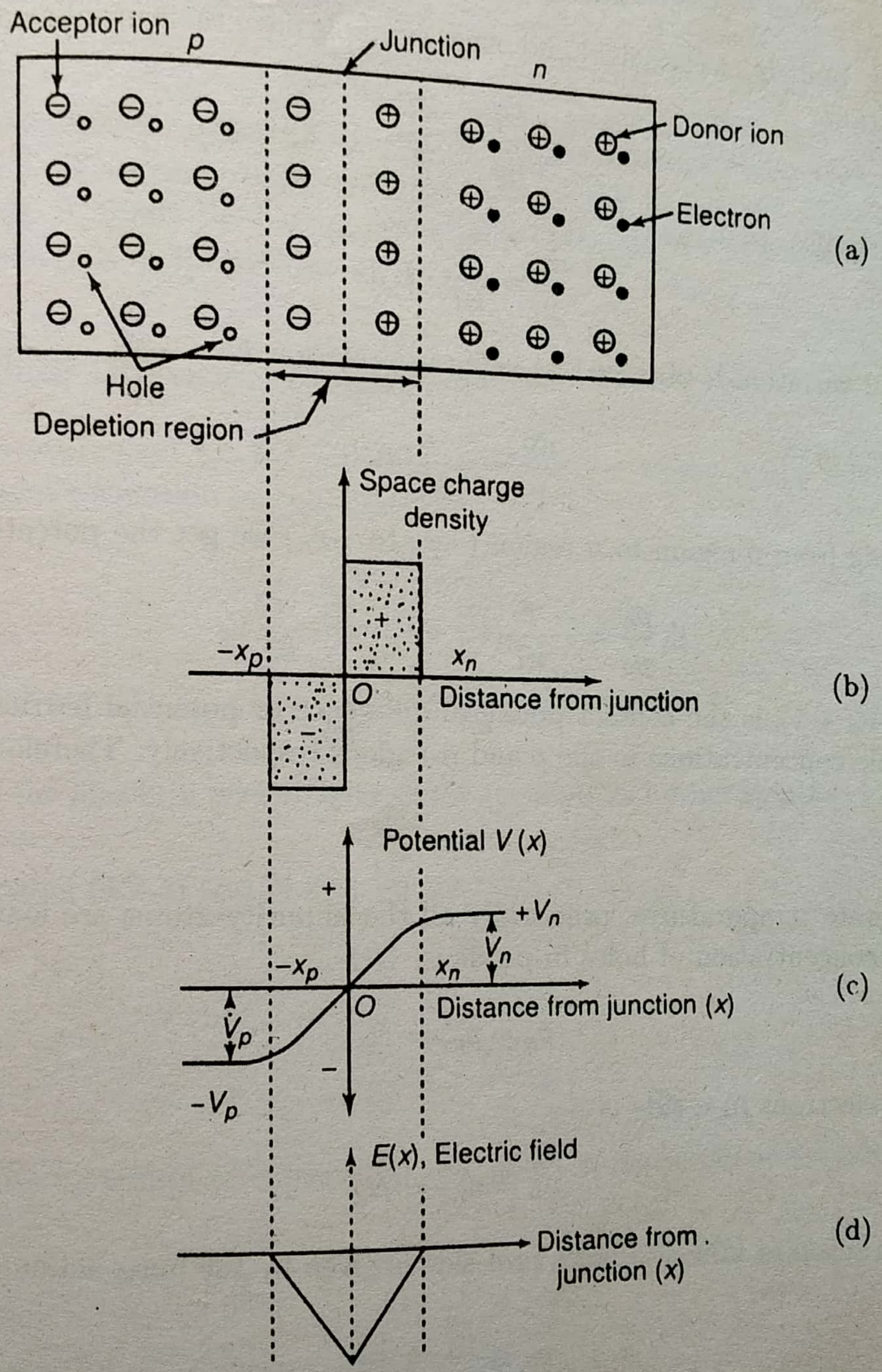


Fig. 4.2-1: Schematic representation of a *p-n* junction (a) Positions of ions and charge carriers (b) variation of space charge density (c) Potential change across the junction; since charge of electron is negative it tends to move from low to high potential region. On the other hand holes tend to move from high to low potential region.

with distance from the junction is shown in Fig. 4.2-1(b). Because of this space charge an intrinsic potential barrier (V_b) develops across the junction. In an open circuited p-n junction net hole and net electron currents must be zero separately. Considering holes only,

$$J_p = ep\mu_p E - eD_p \frac{dp}{dx} = 0 \tag{4.2-1}$$

where E is the built-in electric field and other symbols have their usual meaning. Using Einstein relation

$$\frac{D_p}{\mu_p} = \frac{kT}{e}$$

we get from the Eq. (4.2-1),

$$\frac{dp}{p} = \frac{e}{kT} \cdot E\,dx$$

The potential variation is obtained by using the relation $E = -\frac{dV}{dx}$.

Thus

$$\frac{dp}{p} = -\frac{e}{kT} dV \tag{4.2-2}$$

Integrating from p region to n region ($-x_p$ to $+x_n$) we get the potential rise from p to n regions.

$$\ln \frac{p_n}{p_p} = -\frac{e}{kT}(V_n + V_p) = -\frac{e}{kT} V_b$$

where $V_b = V_n + V_p$ is the contact potential difference or potential barrier; p_p and p_n represent hole concentrations in the p and n-regions respectively. Therefore,

$$p_n = p_p e^{-\frac{eV_b}{kT}} \tag{4.2-3}$$

At moderate temperatures, practically all the impurity atoms are ionised and the equilibrium concentration of holes in p-side is

$$p_p = p_{po} = N_a$$

and that of electrons in n-side is

$$n_n = n_{no} = N_d.$$

The concentration of minority holes in n-side is given by the mass-action law:

$$p_n = \frac{n_i^2}{n_n} = \frac{n_i^2}{N_d}$$

Therefore from Eq. (4.2-3) we get the required height of the potential barrier:

$$V_b = \frac{kT}{e} \ln \frac{p_p}{p_n} = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \tag{4.2-4}$$

The value of V_b for Ge and Si is often in the range 0.5 V to 0.7 V. It depends on the donor and acceptor impurity concentrations, the temperature and the nature of the semiconductor. For example, at room temperature (300 K), $\frac{kT}{e} \approx 0.026$ eV, for Ge with $n_i = 2.5 \times 10^{19}/\text{m}^3$, $N_a = N_d = 10^{22}/\text{m}^3$, the value of V_b is about 0.3 V.

This *barrier potential cannot be measured directly by connecting a voltmeter across the p-n junction.* To show the reading a small amount of current must flow through the circuit. It causes Joule heating in the circuit. Since there in no external source of energy, it must cause simultaneous cooling of the *p-n* junction. But from the principles of thermodynamics it is not possible to derive work by cooling a body below its equilibrium temperature. Thus no current can pass through the circuit and the voltmeter shows zero reading. Actually the barrier potential is balanced by the metal-to-semiconductor contact potentials in the circuit.

B. Width of Depletion Region

To find an expression for the width of the depletion region let us refer to Fig. 4.2-1. From Poisson's equation in one dimension:

$$\frac{d^2V}{dx^2} = \frac{eN_a}{\epsilon} \quad \text{for } x < 0 \tag{4.2-5}$$

$$\frac{d^2V}{dx^2} = -\frac{eN_d}{\epsilon} \quad \text{for } x > 0 \tag{4.2-6}$$

where ϵ is the absolute permittivity of the medium and other symbols have their usual significance.

Integrating (4.2-5) and (4.2-6) we get

$$\frac{dV}{dx} = \frac{eN_a}{\epsilon}x + C_1 \tag{4.2-7}$$

$$\frac{dV}{dx} = -\frac{eN_d}{\epsilon}x + C_2 \tag{4.2-8}$$

To find the constants of integration, C_1 and C_2, we apply the following boundary conditions:

$$\frac{dV}{dx} = 0 \quad \text{at} \quad x = -x_p \quad \text{and} \quad x = x_n$$

Therefore,

$$C_1 = \frac{eN_a}{\epsilon}x_p \quad \text{and} \quad C_2 = \frac{eN_d}{\epsilon}x_n$$

So from Eqs. (4.2-7) and (4.2-8) we get

$$\frac{dV}{dx} = \frac{eN_a}{\epsilon}(x_p + x) \quad \text{and} \quad \frac{dV}{dx} = \frac{eN_d}{\epsilon}(x_n - x) \tag{4.2-9}$$

Integrating again,

$$V = \frac{eN_a}{\epsilon}\left(x_p x + \frac{x^2}{2}\right) + C_3 \quad \text{and} \quad V = \frac{eN_d}{\epsilon}\left(x_n x - \frac{x^2}{2}\right) + C_4$$

To find the constants C_3 and C_4 we apply the boundary conditions:

$$V = -V_p \text{ at } x = -x_p \quad \text{and} \quad V = V_n \quad \text{at} \quad x = x_n$$

Thus we get

$$V = -V_p + \frac{eN_a}{2\epsilon}(x_p + x)^2$$

$$\text{and} \quad V = V_n - \frac{eN_d}{2\epsilon}(x_n - x)^2 \tag{4.2-10}$$

At $x = 0$ both the solutions should give the same value of $\frac{dV}{dx}$ and V. Equating $\frac{dV}{dx}$ at $x = 0$ we get

$$eN_a x_p = eN_d x_n \tag{4.2-11}$$

which is essentially the charge neutrality condition.

Therefore

$$\frac{x_p}{x_p + x_n} = \frac{N_d}{N_a + N_d} \quad \text{and} \quad \frac{x_n}{x_p + x_n} = \frac{N_a}{N_a + N_d} \tag{4.2-12}$$

Equating the values of V at $x = 0$ and using Eq. (4.2-12) we get

$$-V_p + \frac{eN_a}{2\epsilon}x_p^2 = V_n - \frac{eN_d}{2\epsilon}x_n^2$$

Therefore,

$$V_n + V_p = \frac{eN_a}{2\epsilon}x_p^2 + \frac{eN_d}{2\epsilon}x_n^2$$

$$= \frac{e}{2\epsilon}\frac{N_a N_d}{N_a + N_d}(x_p + x_n)^2$$

The width of the depletion layer is

$$W = x_p + x_n = \sqrt{\frac{2\epsilon V_b}{e}\frac{N_a + N_d}{N_a N_d}} \tag{4.2-13}$$

Using Eq. (4.2-4) we can write

$$W = \left[\frac{2\epsilon kT}{e^2}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\ln\frac{N_a N_d}{n_i^2}\right]^{\frac{1}{2}} \tag{4.2-14}$$

The value of W, therefore, depends on the concentrations of impurity atoms on both sides of the p-n junction. Typically its value is of the order of 0.5 μm. It is obvious from Eq. (4.2-14) that the width of the depletion region decreases with increase in doping concentations.

C. Energy Band Diagram

In the n-type semiconductor the Fermi level (E_F) is closer to the conduction band edge E_{cn} while in the p-type material it is closer to the valence band edge E_{vp} (Fig. 4.2-2). When a p-n junction is formed the Fermi level must be the same in both the semiconductors in thermal equilibrium. Otherwise, electrons on one side of the junction would have higher average energy and would transfer to the other lower energy side causing a current flow in an open circuited p-n junction in thermal equilibrium.

Actually, as soon as the junction is formed, electrons from n-side diffuse into p-side and holes from p-side diffuse into n-side. As a result positive space charge appears on the n-side of the junction and negative space charge on the p-side. Such charges raise the position of all the energy levels including Fermi level on p-side and lowers it on the n-side. Ultimately, when the Fermi levels become equal a state of equilibrium is reached with no net current flow through the junction. So the energy band structure of an open circuited p-n junction will be as shown in Fig. 4.2-2. Note that in the energy band diagram we plot the energy of an electron which has a negative charge. Due to charge transfer p-side acquires a negative charge and for this p-bands are slightly raised relative to the n-bands.

Clearly, in equilibrium condition the conduction band edge E_{cp} in the p-side will be at higher level than the conduction band edge E_{cn} in the n-side. Similarly the valence band edge E_{vp} in the p-side will be at higher level than the valence band edge E_{vn} in the n-side (Fig. 4.2-2). Here we have used the subscripts p and n to indicate the p-side and the n-side of the junction.

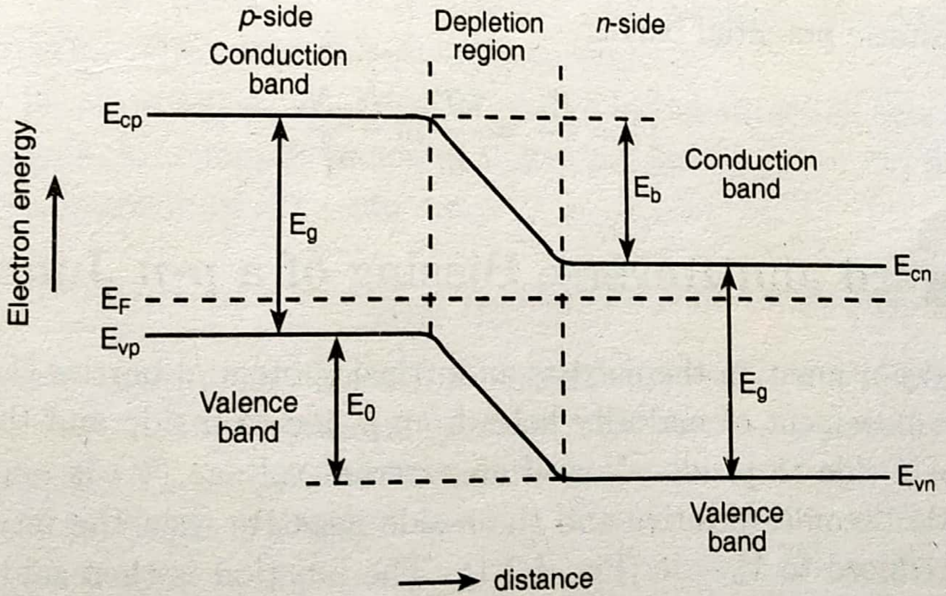


Fig. 4.2-2: Energy band diagram of an open circuited p-n junction

The energy barrier for electrons trying to go from n-side to p-side is

$$E_b = E_{cp} - E_{cn} \tag{4.2-15}$$

From Eqs. (3.5-15) and (3.5-17) we have

$$E_{cn} = E_F + kT \ln \frac{N_c}{N_d} \tag{4.2-16}$$

$$\text{and} \quad E_{vp} = E_F - kT \ln \frac{N_v}{N_a} \tag{4.2-17}$$

Also,

$$E_{cp} = E_{vp} + E_g = E_g + E_F - kT \ln \frac{N_v}{N_a} \tag{4.2-18}$$

Therefore, substituting Eqs. (4.2-16) and (4.2-18) in Eq. (4.2-15), we get

$$E_b = E_g - kT \ln \frac{N_v}{N_a} - kT \ln \frac{N_c}{N_d} \tag{4.2-19}$$

From Eq. (3.5-7)

$$E_g = kT \ln \frac{N_c N_v}{n_i^2} \tag{4.2-20}$$

Therefore,

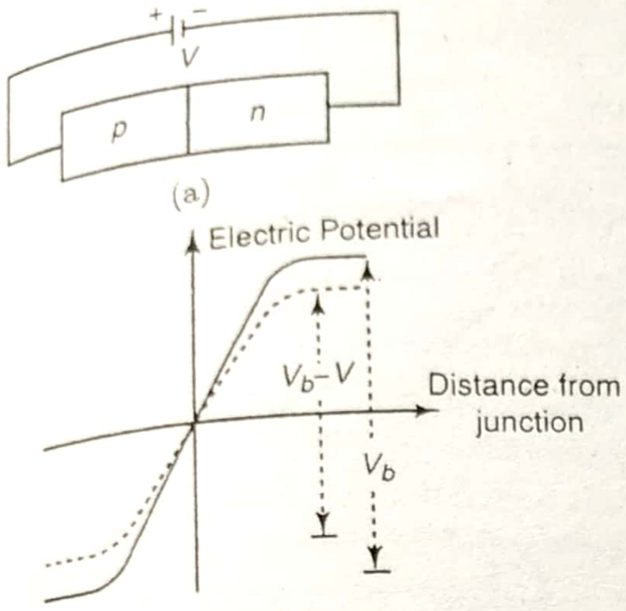
$$\begin{aligned} E_b &= kT \left(\ln \frac{N_c N_v}{n_i^2} - \ln \frac{N_v}{N_a} - \ln \frac{N_c}{N_d} \right) \\ &= kT \ln \frac{N_a N_d}{n_i^2} \end{aligned} \tag{4.2-21}$$

So the electrostatic potential barrier,

$$V_b = \frac{E_b}{e} = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2} \tag{4.2-22}$$

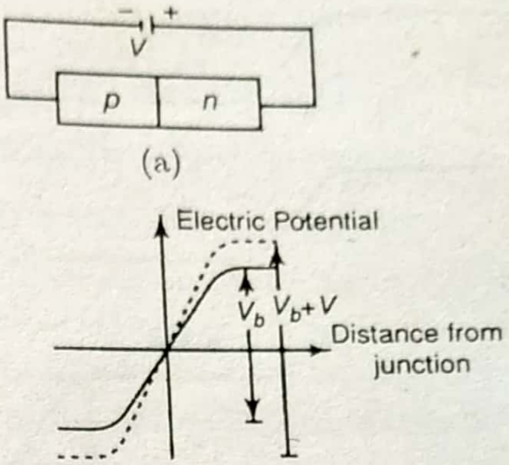
4.3 Forward and Reverse Biasing of a *p-n* Junction

In an unbiased *p-n* junction there exists an intrinsic potential barrier (V_b) which tends to oppose the movement of majority holes from *p*-side to *n*-side and that of majority electrons from *n*-side to *p*-side. Now if an external voltage (V) is applied in such a way that *p*-side becomes positive and the *n*-side negative then the intrinsic potential barrier V_b is reduced to $V_b - V$ (Fig. 4.3-1). The junction is then said to be *forward biased.* Now a good number of holes from *p*-side will be able to overcome this reduced potential barrier and diffuse into *n*-side. Similarly a good number of electrons from *n*-side will be able to cross the reduced barrier and diffuse into *p*-region. These two flows constitute a large current in the same direction, called *forward current* which increases exponentially with the applied voltage. The minority carriers flow down the potential barrier and constitute a small fraction of total current in opposite direction.



(a)

(b)



(a)

(b)

Fig. 4.3-1: (a) Forward biased *p-n* junction
(b) Reduced potential barrier

Fig. 4.3-2: (a) Reversed biased *p-n* junction
(b) Increased potential barrier

If the polarity of the applied voltage (V) is such that p-side becomes negative and n-side positive, the height of the intrinsic potential barrier is increased to $V_b + V$ (Fig. 4.3-2). The junction is then said to be *reverse biased.* The increased barrier height makes the current due to flow of majority carriers negligible. Minority carriers (i.e., electrons in p-side and holes in n-side) move down the potential barrier and hence their motion is not affected by the barrier height. They constitute a small current in the reverse direction, called *reverse saturation* current.

Thus a p-n junction allows easy flow of charge in one direction but restrains the flow in the opposite direction. It indicates that a p-n junction can act as a *rectifying element* for the conversion of a. c. into d.c.

Energy band structure of a biased *p-n* junction and its action as a rectifier

We have seen that in an open circuited p-n junction under thermal equilibrium the energy bands on two sides are separated by an intrinsic barrier E_b [Fig. 4.3-3(a)]. When a forward bias voltage V is applied to the junction, the energy barrier is lowered to the value E_b-eV [Fig. 4.3-3(b)]. The Fermi level on either side of the junction are shifted with respect to each other by an energy eV. The electrons in the conduction band of n-side have certain energy distribution. As a result of the reduction in barrier energy a good number of electrons from the conduction band of n-side can now surmount the reduced barrier and diffuse into p-side where they recombine with majority holes. This gives rise to a large component of diffusion current flowing through the p-n junction. If the minority electrons in the p-side because of their random thermal motion, move close to the junction they are immediately swept down by the energy barrier. This give rise to the electron component of drift current. This is usually small, and insensitive to the barrier height.

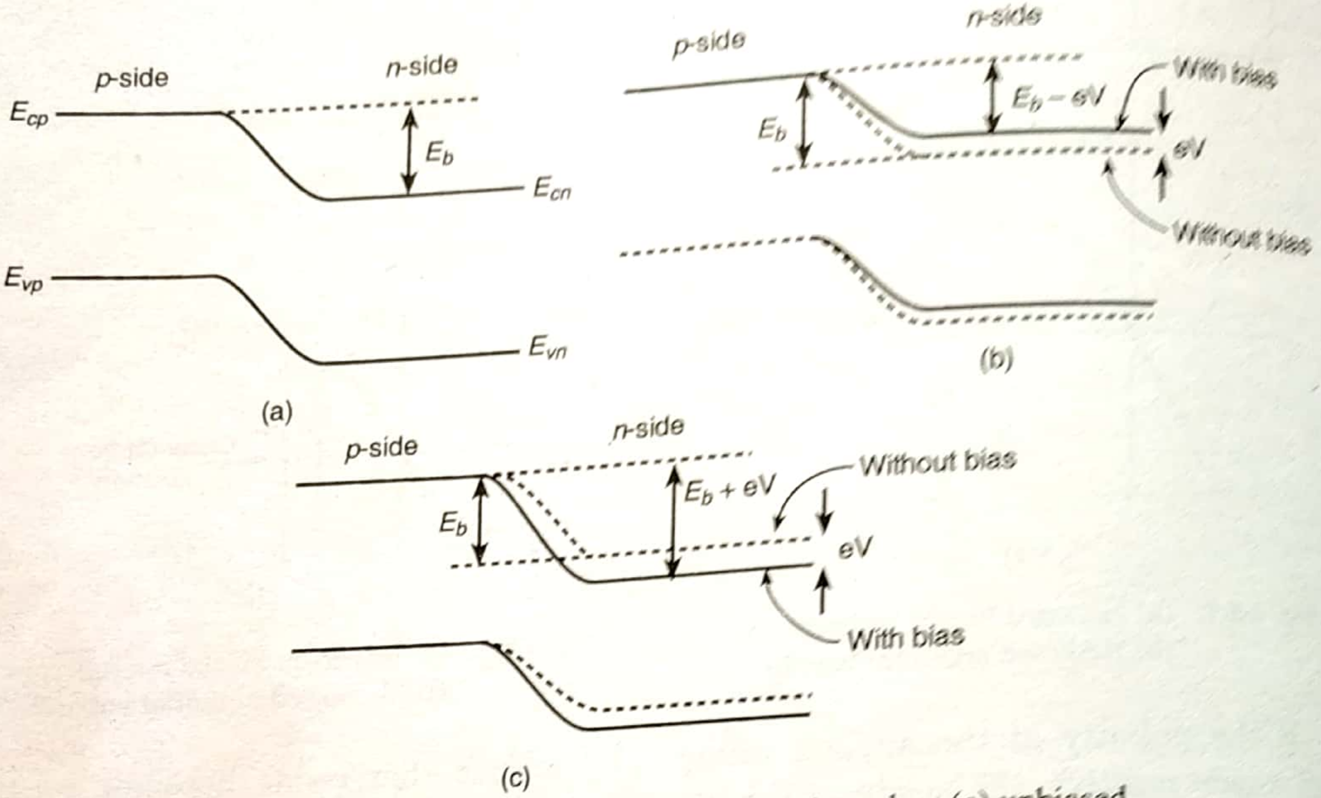


Fig. 4.3-3: Energy band diagram of a *p-n* junction when (a) unbiased (b) forward biased (c) reverse biased

The barrier for electrons is apparent from the energy band diagram which is drawn for electron energies. For holes the energy barriers will be directed oppositely. So a good number of major holes in the valence band of *p*-side can surmount the reduced barrier and diffuse into *n*-side. This gives rise to a large hole component of diffusion current. The minority holes in the *n*-side can easily slide down the barrier and move to the *p*-side. This gives rise to a small hole component of diffusion current. In this way a large current passes through a forward biased *p-n* junction.

When a reverse bias voltage V is applied to the junction the energy barrier is raised to the value $E_b + eV$ [Fig. 4.3-3(c)]. Now very few electrons in the conduction band of *n*-side and holes in the valence band of *p*-side will have enough energy to surmount the increased barrier. Thus diffusion current becomes negligibly small in a reverse biased *p-n* junction. The minority electrons in the conduction band of *p*-side and the minority holes in the valence band of *n*-side flow down the energy barrier and hence their motion is unaffected by the increased barrier. It gives rise to a very small reverse current independent of the applied reverse bias.

Thus current through a forward biased *p-n* junction is very much larger than that for a reverse bias. This means that the junction can be used as a rectifying element to convert a.c. into d.c.

$$I = I_0\left(e^{\frac{eV}{kT}} - 1\right) \tag{4.4-6}$$

where $$I_0 = Ae\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right) \tag{4.4-7}$$

Eq. (4.4-6) is known as *rectifier equation or Shockley equation.*

If the forward voltage V is large compared with $\frac{kT}{e}$ then Eq. (4.4-6) can be approximated as

$$I = I_0 e^{\frac{eV}{kT}} \tag{4.4-8}$$

This shows that the forward current increases exponentially with the applied bias.

For a reverse biased *p-n* junction V is negative. If the magnitude of reverse bias is large compared with $\frac{kT}{e}$ then from the Eq. (4.4-6) we can write

$$I \approx -I_0 \tag{4.4-9}$$

where I_0 is called the *reverse saturation current.* It is independent of reverse voltage but increases with increase in temperature.

In the above derivation we have neglected carrier generation and recombination in the space-charge region. Taking this effect into account small rated current flowing through a *p-n* junction can be approximated by the relation,

$$I = I_0\left(e^{\frac{eV}{\eta kT}} - 1\right) \tag{4.4-10}$$

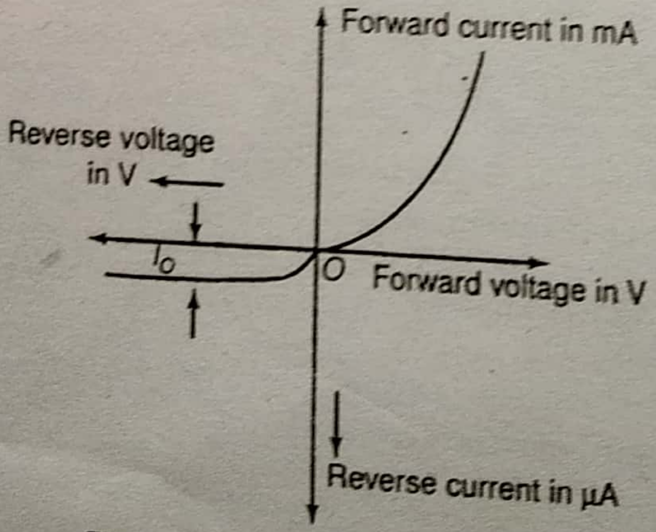


Fig. 4.4-2: Current-voltage characteristic of an ideal *p-n* junction

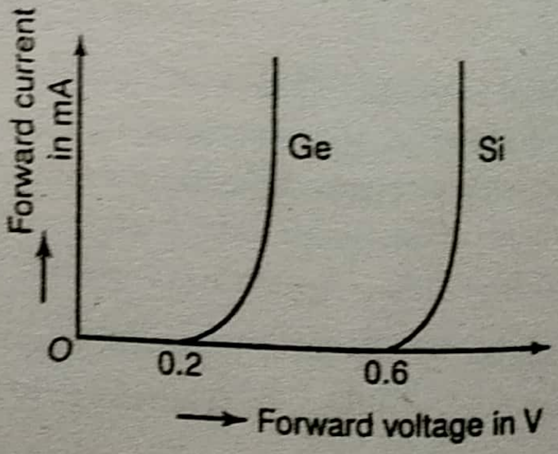


Fig. 4.4-3: Forward characteristics of practical Ge and Si diodes